A state-of-charge based formulation for storage participation in electricity markets

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Abstract

- A market design for storage is proposed that:
  - reflects technical characteristics such as state-of-charge (SOC) and round-trip efficiency,
  - does not require bids and offers by the storage owner within the market horizon,
  - but does require an end-of-horizon bid/offer for deviating its end-of-horizon SOC from its start-of-horizon SOC.
- Focus is on day-ahead markets, but extensions to real-time also mentioned.
- A small day-ahead market example illustrates the market design.
- Future work includes development for real-time and ancillary services markets.

1 Introduction

- Current approach in ISO markets:
  - storage represented as paired (possibly unified) generator and load,
  - offers and bids, respectively, apply to individual intervals over the dispatch horizon, with
  - SOC constraints represented in some cases.
- Fundamental observations:
  - interval-by-interval bids to charge and offers to discharge for storage requires price forecasts, which are subject to:
    - uncertainty due to underlying random events,
    - uncertainty regarding knowledge of system conditions, and
    - endogeneity, since prices forecast by a storage owner need to incorporate the effect of forecasting by other storage.
  - Inherent forecast errors will reduce efficiency of dispatch.
- Interval-by-interval offers for generation and bids for consumption *does not* and *cannot* represent the most salient physical and operational characteristics of storage without relying crucially on a price forecast.

Introduction

- Propose instead that ISO optmimize SOC considering:
  - technical characteristics of storage including SOC, degradation costs, and round-trip efficiency, and
  - with a single bid/offer from storage for deviating the end-of-horizon SOC from the start-of-horizon SOC.
- CAISO, NYISO, and PJM (for pumped-storage hydro) have models that represent SOC across horizon, but in contrast CAISO and NYISO include interval-by-interval bids and offers and none include an end-of-horizon bid/offer.
- Formulation follows literature for pumped hydro storage.
- Focus here on power and energy in day-ahead (DA), but recognize that real-time (RT) and ancillary services (AS) must be treated.

2 Proposed model of storage

2.1 Dispatch intervals

- Interval index t: hourly for DA; 5-minute for RT,
- Dispatch horizon is $t = 1, \ldots, T$,
- Interval $t = 0$ is immediately prior to the beginning of the horizon.

2.2 Charging and generation variables

- Charging power at end of interval t is $p_t \in \mathbb{R}$.
- Discharging power at end of interval t is $g_t \in \mathbb{R}$.
- In DA, power level applies from just after the beginning of the interval and throughout the rest of the interval.
- In RT, power levels are assumed to ramp within intervals.
- Collect charging and discharging variables for $t = 1, \ldots, T$ into vectors $p, g \in \mathbb{R}^T$.

2.3 Charging and discharging constraints

$$0 \leqslant p_t \leqslant \overline{p}, \quad \forall t = 1, \ldots, T, \tag{1}$$
$$0 \leqslant g_t \leqslant \overline{g}, \quad \forall t = 1, \ldots, T, \tag{2}$$
$$p_t \perp g_t, \quad \forall t = 1, \ldots, T, \tag{3}$$

- "$\perp$" in (3) denotes that the variables on either side of it cannot both be positive.

2.4 State-of-charge

- $s_t \in \mathbb{R}$ is SOC at the end of interval t.
- Assumed known initial SOC just prior to the beginning of the horizon is $s_0 \in \mathbb{R}$.
- Linear model of conversion losses for charging and discharging and self-discharge of the storage (for DA; case of RT is more complex):

$$s_t = \Delta T \alpha p_t - \Delta T \beta g_t + \gamma s_{t-1}, \forall t = 1, \ldots, T, \tag{4}$$

- where:
  - ΔT is length of the interval;
  - coefficients α and β satisfy $\beta > \alpha > 0$ and round-trip efficiency $\alpha/\beta < 1$; and,
  - $0 < \gamma \leqslant 1$ is per interval self-discharge.
- For DA, define $\overline{\overline{s}}_T = (\gamma)^T s_0$, the SOC at the end of the DA horizon if the storage were to neither charge nor discharge during the horizon,
- For RT, define $\overline{\overline{s}}_T$ to be the interpolation of DA SOC for the end of RT horizon, which is SOC at the end of the RT horizon if the storage followed the DA positions in RT.

2.5 SOC DA example

- Let $s_0 = 100\text{MWh}$, $\gamma = 0.99$, and $T = 24$.
- Then $\overline{\overline{s}}_T = (0.99)^{24} \times 100\text{MWh} = 78.6\text{MWh}$.
- $\overline{\overline{s}}_T$ is the reference for considering offers and willingness-to-pay by the storage for deviating its end-of-horizon SOC from (the implications of) its beginning-of-horizon SOC.
- $\overline{\overline{s}}_T$ would be the end-of-horizon SOC if the storage did not participate in the market.

2.6 State-of-charge limits

- Maximum and minumum SOC $\overline{s}$ and $\underline{s}$:

$$\begin{aligned} \underline{s} \quad &\leqslant \quad s_0 \quad \leqslant \quad \overline{s}, && (5) \\ \gamma s_{t-1} + \Delta T \alpha p_t \quad &\leqslant \quad \overline{s}, \forall t = 1, \ldots, T, && (6) \\ \gamma s_{t-1} - \Delta T \beta g_t \quad &\geqslant \quad \underline{s}, \forall t = 1, \ldots, T. && (7) \end{aligned}$$

2.7 Other joint constraints on power and SOC

- "Foldback" can be represented with additional linear constraints.

2.8 Storage degradation costs

- Linear cost function for degradation of storage:

$$\epsilon \mathbf{1}^\dagger p + \zeta \mathbf{1}^\dagger g, \tag{8}$$

- where:
  - $\mathbf{1}$ is the vector of all ones,
  - † denotes transpose, and
  - $\epsilon, \zeta \in \mathbb{R}_+$ are coefficients representing degradation costs per unit time.

2.9 Storage bid

- Bid for deviating final SOC from $\overline{\overline{s}}_T$, specified by (net) willingness-to-pay $w : \mathbb{R} \to \mathbb{R}$:
  - storage is willing-to-pay (or receive if negative) an amount $w\big(s_T - \overline{\overline{s}}_T\big)$ if final SOC is equal to s_T.
  - w is monotonically non-increasing, providing a demand curve for SOC,
  - $b : \mathbb{R} \to \mathbb{R}$ is the definite integral of w between 0 and $\big(s_T - \overline{\overline{s}}_T\big)$,
  - b is concave and $b(0) = 0$.
- b is the bid benefit of deviating the end-of-horizon storage SOC from the value, $\overline{\overline{s}}_T$, that the SOC would have been absent market participation over the horizon.
- Benefit b represents value to storage of accumulating a net increase in SOC over the horizon:
  - not equivalent to value of charging in the last interval of the horizon or the value of charging in any other particular interval of the horizon.
- Settlement is based only on energy charged and discharged and not directly on the SOC:
  - no direct payment for SOC.

2.10 Bid example

- Net willingness-to-pay \$40/MWh for all charging and discharging,
- then $b(\Delta s_T) =$ \$40/MWh $\times\ \Delta s_T, \forall \Delta s_T$,
- If final SOC exceeds $\overline{\overline{s}}_T$ by $\Delta s_T = 10$MWh,
- then bid benefit is \$400.
- ISO considers both degradation costs in (8) and the bid benefit b of deviating the end-of-horizon storage SOC from $\overline{\overline{s}}_T$:
  - settlement is based only on energy charged and discharged, but
  - the willingness-to-pay can set the price (see in example).

2.11 Balance of market

- Vector $x \in \mathbb{R}^N$ for variables defining rest of the market over the horizon,
- Offer net cost function for the rest of the market $c^\dagger x$, with coefficients $c \in \mathbb{R}^N$ representing costs per unit time,
- Constraints on rest of the market $Cx \leqslant d$, including both the *system* constraints such as transmission constraints and the *private* constraints such as generation limits:
  - matrix C and vector d have suitable dimensions.
- Ignore any binary variables required for unit commitment.

2.12 Market formulation and pricing model

- ISO problem:

$$\min_{\substack{x \in \mathbb{R}^N, \\ p,g,s \in \mathbb{R}^T}} \left\{ \begin{array}{c} \Delta T(c^\dagger x) \\ + \Delta T(\epsilon \mathbf{1}^\dagger p + \zeta \mathbf{1}^\dagger g) \\ - b(s_T - \overline{\overline{s}}_T) \end{array} \middle| \begin{array}{c} (1) - (7), \\ Cx \leqslant d \end{array} \right\}. \tag{9}$$

- Minimizes the sum of:
  - offered net costs of the rest of the market,
  - degradation costs of the storage,
  - minus bid benefits of deviating the end-of-horizon storage SOC from the value that the SOC would have absent market participation.
- Convex, ignoring binary variables, and maximizes (revealed) welfare.
- LMPs defined in the usual way as depending on the Lagrange multipliers of system constraints.
- No direct payment of storage SOC:
  - discharging is paid at the LMP and charging pays at the LMP.

2.13 "Storage surplus"

- Definition:
  - revenue from discharging,
  - plus bid benefit of deviation of end-of-horizon SOC,
  - minus payments for charging,
  - minus degradation costs.
- Represents net payoff to storage based on participating in market, considering degradation costs and the value to storage of accumulating a net increase in SOC over the horizon.
- With usual argument for convex markets, the storage surplus is non-negative.
- If energy arbitrage is valuable then storage surplus will be strictly positive.
- See in example in Section 3.

2.14 Discussion

- Storage benefits from arbitrage opportunities without predicting prices within the horizon:
  - no forecast of prices within the horizon since ISO *calculates* the prices.
- Storage benefits *are* necessarily based on forecast of market prices beyond the end-of-horizon:
  - prices after the end of the dispatch horizon can only be forecast imperfectly by the ISO,
  - while risks due to any forecast errors will generally be borne by the storage.
- Forecasts beyond the end-of-horizon, and the risks due to forecast errors, are the legitimate and appropriate prerogative of the storage, particularly in the DA.
- Argument is weaker in RT where ISO may have better forecasts than market participants past the end of the RT lookahead horizon.
- Forecast burden for storage is lowered compared to current market models since only forecasts of prices beyond the end-of-horizon are required.

3 Example

- Simplified DA market with $T = 6$ intervals each of duration $\Delta T = 4$ hours:
- Demand fixed for each interval, so maximizing welfare equivalent to minimizing:
  - thermal costs,
  - plus storage degradation costs,
  - minus the benefits of deviating the end-of-horizon storage SOC.
- Thermal generation, renewable generation, and storage each represented as a stylized single aggregate.
- No reserves nor other AS are modeled.
- No unit commitment.
- No transmission.

3.1 Generation and storage characteristics

- Thermal:
  - offer price \$50/MWh in all intervals,
  - minimum production 0MW, and
  - maximum production 100MW.
- Renewable:
  - offer price \$0/MWh for all available capacity,
  - minimum production 0MW, and
  - maximum production equal to “renewable available capacity.”
- Storage:
  - $\alpha = 0.9, \beta = 1.0$, so roundtrip efficiency 0.9,
  - $\gamma = 1$, so no self-discharge,
  - $\epsilon =$ \$1/MWh, $\zeta =$ \$1/MWh, so modest degradation costs,
  - $\overline{p} =$ 200MW and $\overline{g} =$ 200MW, and
  - $\underline{s} = s_0 =$ 0MWh, so zero SOC initially.
  - maximum SOC $\overline{s} =$ 1200MWh,
  - willingness-to-pay for increasing the end-of-horizon SOC compared to $s_0 =$ 0MWh is \$40/MWh.

3.2 Demand and renewable available capacity

Interval t	1	2	3	4	5	6
Demand (MW)	100	200	300	300	200	100
Renewable Available Capacity (MW)	200	100	500	500	25	0

- Stylized typical daily demand pattern:
  - relatively low from midnight to 4am (interval 1),
  - increasing demand through noon,
  - continued high demand from noon to 4pm (interval 4),
  - decreasing demand in evening through midnight.
- Stylized typical renewable available capacity pattern:
  - onshore wind peaking sometime after midnight,
  - low available capacity in early morning from 4am to 8am,
  - maximum available capacity of solar between 8am and 4pm,
  - rapid decline in available capacity of renewables until midnight.
- Two potential opportunities per day for storage charging and discharging.

3.3 Results

Interval t	1	2	3	4	5	6
Renewable Production (MW)	200	100	500	433	25	0
Thermal Production (MW)	0	10	0	0	0	0
Storage Discharge (MW)	0	90	0	0	175	100
Storage Charging (MW)	100	0	200	133	0	0
Storage SOC (MWh)	360	0	720	1200	500	100
Price ($/MWh)	0	50	0	0	41	41

- Thermal generation offer sets the price only in interval 2.
- Storage charged and discharged in intervals 1 and 2, respectively, with state-of-charge zero at the end of interval 2.
- Renewable generation not fully utilized in interval 4, driving price to zero.
- Price also zero in interval 3, even though the renewables are fully deployed, because the maximum SOC limit of $\overline{s} = 1200$MWh limits the total charging in intervals 3 and 4.
- End-of-horizon SOC increased from s_0 to $s_T = 100$MWh.
- Storage did not need to forecast prices within horizon.

3.4 Discussion

- Range of market clearing prices in interval 1 between:
  - $0/MWh (offer price of the renewable resource) and
  - $43.1/MWh (maximum price in interval 1 that results in storage being competitive with thermal generation in interval 2).
- Range of market clearing prices in intervals 5 and 6 between:
  - $41/MWh (storage willingness-to-pay plus the discharging degradation cost) and
  - $50/MWh (offer price of thermal resource).
- Storage surplus of $63,907 (using low end of ranges of market clearing prices) due to:
  - revenue from discharging, $63,100,
  - plus bid benefit of deviation of end-of-horizon SOC, $4,000,
  - minus payments for charging, $0,
  - minus degradation costs, $3193.

3.5 Interval-by-interval bids/offers

- A bid/offer in each interval for an arbitrary quantity of charging or discharging at a fixed price would result in welfare maximization for any fixed price in the range from $0/MWh to $50/MWh.
- But fixed price interval-by-interval bids/offers would result in negative storage surplus!
- For positive surplus, interval-by-interval bids and offer prices need to be closer to the interval-by-interval clearing prices:
  - storage would need a better forecast of the prices for interval-by-interval bids and offers.

4 Conclusion

- Described a model for storage participation in electricity markets that explicitly represents and optimizes storage SOC considering round-trip efficiency and other technical characteristics of storage.
- Implementation would require changes to the structure of existing electricity markets.
- Several issues remain including the design of AS markets.